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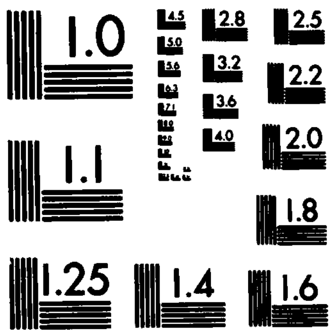

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REFRACTION EQUATIONS FOR HYDRONS CONSIDERING BOTTOM TOPOGRAPHY AND CURRENTS

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BY J. ERNEST BREEDING, JR.

TECHNICAL REPORT NO. TR-JEB-5
DEPARTMENT OF OCEANOGRAPHY AND OCEAN ENGINEERING
FLORIDA INSTITUTE OF TECHNOLOGY

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ABSTRACT

Refraction equations are derived for hydrons (wave packets) in which the water depth and current are both variable. A comparison is made of the refraction equations obtained by different investigators.

1. Introduction

The main purpose in this paper is to derive the refraction equations for surface water waves which are moving in a variable current over a variable bottom topography. A second purpose in the paper is to compare the refraction equations obtained by various investigators. Although the results are presented for the example of water waves, the refraction expressions obtained are applicable to all kinds of waves.

The waves are presumed to propagate in the form of wave packets. A wave packet is obtained by the summation of all sine waves over a narrow range of both frequencies and directions. Synge (1962) has suggested the name hydron for wave packets associated with water waves. It is assumed that wave properties such as the wave speed, direction, and wavelength do not change much over distances of one wavelength. Ray curvature expressions are

derived following the procedure outlined by Laudau and Lifshitz (1959). The water wave velocities are assumed to be much greater in magnitude than the currents, and terms higher than first order in the current velocity are ignored.

2. Hydrons in currents

An unprimed coordinate system xy is fixed in space on the water surface. A primed coordinate system x'y' is considered to be stationary with respect to a water current and is moving with velocity $\vec{u}$ relative to the xy coordinate system. The position vector $\vec{r}$ in the xy coordinate system and the position vector $\vec{r}'$ in the x'y' coordinate system are related by

$$\vec{r} = \vec{u}t + \vec{r}' \tag{1}$$

where t is time.

For an observer moving with the current a hydron is expressed by (Breeding, 1978)

$$\eta = e^{i(\vec{k}\cdot\vec{r}' - \omega' t)} \int_{\Omega-\epsilon}^{\Omega+\epsilon} A(\omega)\, e^{i(\vec{m}\cdot\vec{r}' - \sigma' t)}\, d\omega \tag{2}$$

where η is the displacement, A is the amplitude, $\vec{k}$ is the wave number, and ω is the radian frequency of the wavelets. The wave number $\vec{m}$ and the radian frequency σ of the wave packet are defined by

$$\vec{m} = \Delta\vec{k} \tag{3}$$

$$\sigma = \Delta\omega \tag{4}$$

The average radian frequency of the wave packet is denoted by Ω and the band-width is 2ϵ.

It is important to note the distinction between δk and Δk.

$$\delta k = |\vec{k} + \Delta\vec{k}| - |\vec{k}| \tag{5}$$

The differential δk depends only on the difference in the magnitudes of the component wave numbers. The differential Δk depends on both the difference in the magnitudes of the component wave numbers and their directions. The directions of both $\vec{k}$ and $\delta\vec{k}$ with respect to the positive x-axis is γ. The direction of $\Delta\vec{k}$ with respect to the positive x-axis is denoted by θ. It can be shown (Breeding, 1978) that

$$\delta k = \Delta k \cos\phi \tag{6}$$

where

$$\phi = \theta - \gamma \tag{7}$$

The velocities of the wavelets and packets relative to the current are determined by holding the phases constant, respectively, in (2). The wavelet (phase) velocity is defined by

$$\vec{v}' = \frac{\omega'}{k}\hat{e}_k \tag{8}$$

where $\hat{e}_k$ is a unit vector in the direction of $\vec{k}$. The velocity of the wave packets is called the geometric group velocity (Breeding, 1978), and it is given by

$$\vec{G}' = \frac{\partial\omega'}{\partial k}\cos\phi\,\hat{e}_m \tag{9}$$

where $\hat{e}_m$ is a unit vector in the direction of $\Delta\vec{k}$.

When (1) is solved for $\vec{r}'$ and the result is substituted into (2) the displacement is expressed by

$$\eta = e^{i[\vec{k}\cdot\vec{r} - (\omega' + \vec{k}\cdot\vec{u})t]} \int_{\Omega-\epsilon}^{\Omega+\epsilon} A(\omega)\, e^{i[\vec{m}\cdot\vec{r} - (\sigma' + \vec{m}\cdot\vec{u})t]}\, d\omega \tag{10}$$

From (10) it is seen that the frequencies in the fixed coordinate system are defined by

$$\omega = \omega' + \vec{k}\cdot\vec{u} \tag{11}$$

$$\sigma = \sigma' + \vec{m}\cdot\vec{u} \tag{12}$$

Despite the difference in frequencies, the wavelengths $\lambda = 2\pi/k$ and $L = 2\pi/m$

are the same in both the xy and x'y' coordinate systems. The velocities of the wavelets and wave packets in the xy coordinate system are found by taking the time derivative of (1) and substituting, respectively, (8) and (9) to obtain

$$\vec{v} = \vec{v}' + \vec{u} \tag{13}$$

$$\vec{G} = \vec{G}' + \vec{u} \tag{14}$$

where $v' = \omega'/k$ and $G' = (\partial\omega'/\partial k)\cos\phi$. If the current vanishes (13) and (14) with definitions (8) and (9) are the same as obtained by Breeding (1978) for the velocities of the wavelets and wave packets, respectively.

3. Snell's laws for hydrons for refraction due to bottom topography and currents

Wave refraction is considered for the idealized case where a current is directed parallel to a parallel set of water depth contours. Both the water depth and the magnitude of the current vary in the lateral direction. The refraction of the wavelets is illustrated in Figure 1. A discontinuity in both the water depth and current magnitude are indicated. The subscript i denotes the incident wave while the subscript t depicts the transmitted (refracted) wave.

The wavelet crest must be continuous across the boundary. Therefore, as the incident wave crest advances one wavelength the refracted wave crest also advances one wavelength. With respect to the fixed coordinate system the time taken for a crest to advance one wavelength is the same as the incident wave period on both sides of the boundary. However, relative to the currents the wave period changes as the waves cross the boundary.

The angle of refraction is determined from the condition that the com-

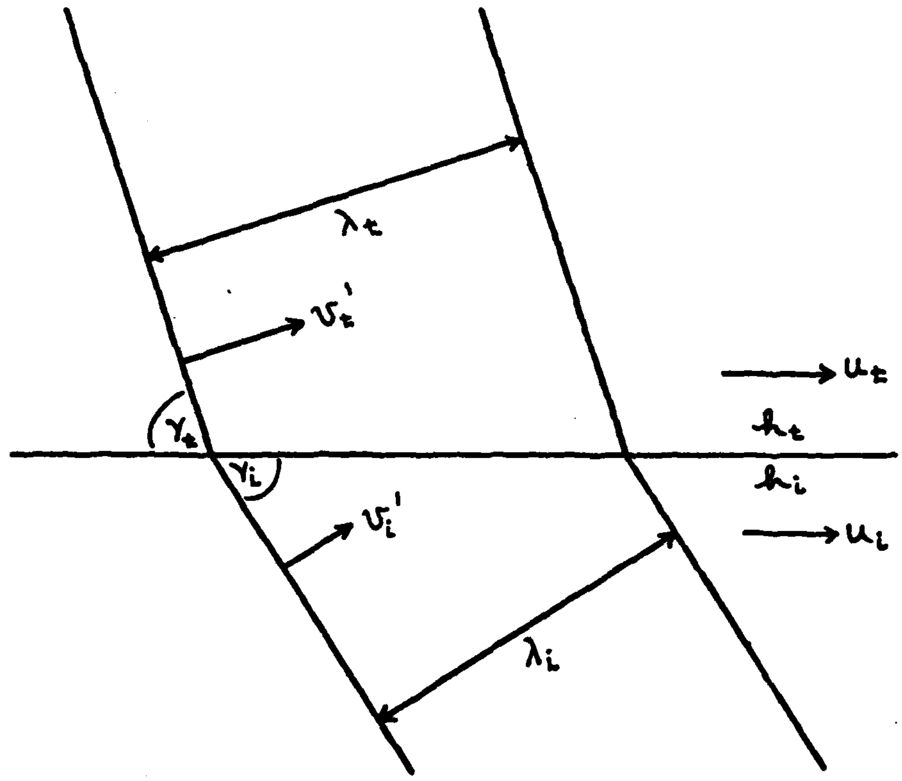

Figure 1. Continuity of wave crest across a discontinuity in water depth and current speed.

ponent of the wave number parallel to the boundary is the same on both sides of the boundary. The result is Snell's law

$$k \sin \gamma = \text{constant} \tag{15}$$

The angle γ is the angle the crest makes with the boundary or alternatively the angle between the orthogonal to the crest and the normal to the boundary. Since ω remains constant in the fixed coordinate system (15) can be divided by ω to obtain

$$\frac{\sin \gamma}{v^*} = \text{constant} \tag{16}$$

where with appropriate subscripts

$$v^* = v' + u_k \tag{17}$$

The phase speed v' is defined relative to the current, and u_k is the component of the current in the direction of $\vec{k}$. The component u_k is given by

$$u_k = u \sin \gamma \tag{18}$$

The refraction law can be stated

$$\frac{\sin \gamma_t}{v_t' + u_t \sin \gamma_t} = \frac{\sin \gamma_i}{v_i' + u_i \sin \gamma_i} \tag{19}$$

If $u_i = 0$ this result reduces to the one obtained by Johnson (1947).

The refraction law for hydrons is obtained in the same manner as is the wavelet refraction law. A crest determined by the interference maximum of the hydron (alternatively, the nodal line between wave packets can be considered) must be continuous across a boundary. Snell's law for the hydron is

$$m \sin \theta = \text{constant} \tag{20}$$

where θ is the direction of $\vec{m}$ with respect to the normal to the boundary. In the fixed coordinate system σ is constant on both sides of the boundary. When (20) is divided through by σ the result is

$$\frac{\sin \theta}{G^*} = \text{constant} \tag{21}$$

where the differential forms of (4) and (6) have been used. Further

$$G^* = G' + u_m \tag{22}$$

where G' is the speed of the hydron relative to the current and $u_m = u \sin\theta$ is the component of the current in the direction of $\vec{m}$. The hydron refraction law, which is the counterpart of (19), can be stated

$$\frac{\sin\theta_t}{G_t' + u_t \sin\theta_t} = \frac{\sin\theta_i}{G_i' + u_i \sin\theta_i} \tag{23}$$

where G' is defined by (9).

In the absence of any currents (23) reduces to the results obtained by Stoneley (1935) and Breeding (1978). Stoneley used the result to determine the direction of amplitude (group) fronts along monochromatic trajectories, whereas Breeding used the result to define wave packet trajectories.

4. Ray theory and Hamilton's equations

There is a very useful analogy between ray theory and particle mechanics. Synge (1962) has used the dispersion relation for water waves to derive an equation which is an analogue of the Hamilton-Jacobi equation used to solve mechanical problems. Goldstein (1950), Landau and Lifshitz (1959), and Lindsay (1960) discuss the similarity between the eikonal equation for rays and the Hamilton-Jacobi equation. In what follows the ray theory treatment presented in articles 66 and 67 of Landau and Lifshitz (1959) for nondispersive waves will be extended to dispersive waves.

A wave packet has two phases. The phase ψ of the wavelets can be stated

$$\psi = k_x x + k_y y - \omega t \tag{24}$$

The wave number $\vec{k}$ and frequency ω can be defined in terms of ψ.

$$\vec{k} = \nabla\psi \tag{25}$$

$$\omega = -\frac{\partial\psi}{\partial t} \tag{26}$$

By definition $k^2 = k_x^{\ 2} + k_y^{\ 2} = \frac{\omega^2}{v^2}$. Thus, substitutions for k_x, k_y, and ω yield

$$\left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial y}\right)^2 - \frac{1}{v^2}\left(\frac{\partial\psi}{\partial t}\right)^2 = 0 \tag{27}$$

Additionally, the wave packet has the phase Υ where

$$\Upsilon = m_x x + m_y y - \sigma t \tag{28}$$

Further

$$\vec{m} = \nabla\Upsilon \tag{29}$$

$$\sigma = -\frac{\partial\Upsilon}{\partial t} \tag{30}$$

Since $m^2 = m_x^{\ 2} + m_y^{\ 2} = (\sigma^2/G^2)$ it is seen that

$$\left(\frac{\partial\Upsilon}{\partial x}\right)^2 + \left(\frac{\partial\Upsilon}{\partial y}\right)^2 - \frac{1}{G^2}\left(\frac{\partial\Upsilon}{\partial t}\right)^2 = 0 \tag{31}$$

Equations (27) and (31) are first order partial differential equations which are similar to the Hamilton-Jacobi equation. In mechanics S is the action of a particle. The momentum $\vec{p} = \nabla S$. Hamilton's function H is the energy of a particle and $H = -\partial S/\partial t$. The solution of the Hamilton-Jacobi equation is equivalent to solving Hamilton's equations

$$\frac{d\vec{p}}{dt} = -\nabla H = -\frac{\partial H}{\partial \vec{r}} \tag{32}$$

$$\vec{V} = \frac{d\vec{r}}{dt} = \nabla_{p} H = \frac{\partial H}{\partial \vec{p}} \tag{33}$$

where in (33) $\vec{V}$ is the particle velocity. In ray theory the phases ψ and Υ are analogous to S. The wave numbers $\vec{k}$ and $\vec{m}$ are analogous to $\vec{p}$, and ω and σ are analogous to H. Hamilton's ray equations can be expressed as

$$\frac{d\vec{k}}{dt} = -\nabla\omega = -\frac{\partial\omega}{\partial\vec{r}} \tag{34}$$

$$\frac{d\vec{m}}{dt} = -\nabla\sigma = -\frac{\partial\sigma}{\partial\vec{r}} \tag{35}$$

$$\vec{G} = \frac{d\vec{r}}{dt} = \frac{\partial\omega}{\Delta\vec{k}} = \frac{\partial\omega}{\partial k}\cos\phi\,\hat{e}_m \tag{36}$$

In (34) $\vec{k}$ and $\vec{r}$ are independent variables whereas in (35) $\vec{m}$ and $\vec{r}$ are independent variables. In the absence of currents ω is constant along a ray (Landau and Lifshitz, 1959). Equation (36) is the velocity of the moving

interference pattern which constitutes the wave packet. If there are currents (36) becomes the same as (9).

Equation (34) can be derived on the assumption that there is conservation of the wave crests (Phillips, 1977). The partial derivative of (25) is taken with respect to time and (26) is used. Since $\vec{k}$ and $\vec{r}$ are independent variables $d\vec{k}/dt = \partial\vec{k}/\partial t$. The result is (34). The one-dimensional form of this equation was derived by Rossby (1945). In a similar fashion (29) and (30) can be combined to obtain (35).

5. Ray curvature formulas for wavelets, packets, and rays

Ray curvature expressions will be developed based on the method presented in articles 66 and 67 by Landau and Lifshitz (1959). It is assumed that v, G, and u are functions of x,y. In addition $u \ll v$, $u \ll G$, and terms higher than first order in $\vec{u}$ are neglected.

5.1 Ray curvature for wavelets. Equation (11) is substituted into (34) to obtain

$$\frac{d\vec{k}}{dt} = -k\nabla v' - \nabla(\vec{u}\cdot\vec{k}) \tag{37}$$

where $v' = \omega'/k$. It is convenient to employ the vector identity

$$\nabla(\vec{u}\cdot\vec{k}) = \vec{k}\cdot\nabla\vec{u} + \vec{k}\times(\nabla\times\vec{u}) + \vec{u}\cdot\nabla\vec{k} + \vec{u}\times(\nabla\times\vec{k}) \tag{38}$$

The last two terms on the right hand side of (38) are zero since $\vec{k}$ and $\vec{r}$ are independent variables. Therefore

$$\frac{d\vec{k}}{dt} = -k\nabla v' - \vec{k}\cdot\nabla\vec{u} - \vec{k}\times(\nabla\times\vec{u}) \tag{39}$$

An alternative expression can be derived for $d\vec{k}/dt$. Let the unit vector $\hat{e}_k = \vec{k}/k$. Then

$$\frac{d\vec{k}}{dt} = k\frac{d\hat{e}_k}{dt} + \hat{e}_k\frac{dk}{dt} \tag{40}$$

when (39) and (40) are combined and the result simplified it is found that

$$\frac{d\hat{e}_k}{dt} = -\nabla v' - \hat{e}_k \cdot \nabla \vec{u} - \hat{e}_k \times (\nabla \times \vec{u}) - \hat{e}_k \frac{1}{k}\frac{dk}{dt} \tag{41}$$

Since $\hat{e}_k \cdot \hat{e}_k = 1$ it follows that

$$\hat{e}_k \cdot \frac{d\hat{e}_k}{dt} = 0 \tag{42}$$

Thus $\hat{e}_k$ and $d\hat{e}_k/dt$ are perpendicular. The unit vector $\hat{n}_k$ is taken tangent to the wavelet crest, i.e., perpendicular to $\hat{e}_k$. Then

$$\frac{d\gamma}{dt} = \hat{n}_k \cdot \frac{d\hat{e}_k}{dt} = -\hat{n}_k \cdot \left[\nabla v' + \hat{e}_k \cdot \nabla \vec{u} + \hat{e}_k \times (\nabla \times \vec{u})\right] \tag{43}$$

where γ denotes the direction of $\vec{k}$ with respect to the positive x-axis.

Equation (43) simplifies to

$$\begin{aligned} \frac{d\gamma}{dt} = {} & \sin\gamma \frac{\partial v'}{\partial x} - \cos\gamma \frac{\partial v'}{\partial y} + \sin^2\gamma \frac{\partial u_y}{\partial x} + \\ & + \sin\gamma \cos\gamma \left(\frac{\partial u_x}{\partial x} - \frac{\partial u_y}{\partial y}\right) - \cos^2 \frac{\partial u_x}{\partial y} \end{aligned} \tag{44}$$

If v' is constant (44) reduces to Zermelo's result reported by Arthur (1950).

The component of the current in the direction of $\vec{k}$, denoted by u_k, is given by

$$u_k = \hat{e}_k \cdot \vec{u} \tag{45}$$

It can be shown that (44) is equivalent to

$$\frac{d\gamma}{dt} = \sin\gamma \frac{\partial}{\partial x}(v' + u_k) - \cos\gamma \frac{\partial}{\partial y}(v' + u_k) \tag{46}$$

This result was obtained by Arthur (1950).

The arc length along the trajectory of an orthogonal to a wavelet crest is defined by $ds_k = (v' + u_k)\, dt \approx v'\, dt$. As a result (46) becomes

$$\frac{d\gamma}{ds_k} = \frac{1}{v'}\left[\sin\gamma \frac{\partial}{\partial x}(v' + u_k) - \cos\gamma \frac{\partial}{\partial y}(v' + u_k)\right] \tag{47}$$

This expression defines the ray curvature of the wavelets, and is a differential form of Snell's law (19). If there are no currents (47) reduces to the expression derived by Munk and Arthur (1952) and Arthur <u>et al.</u> (1952).

5.2 Ray curvature for wave packets. The ray curvature expression for the hydrons is derived in the same manner as is the ray curvature expression for the wavelets. It is found that

$$\frac{d\theta}{dt} = \sin\theta \frac{\partial}{\partial x}(G' + u_m) - \cos\theta \frac{\partial}{\partial y}(G' + u_m) \tag{48}$$

where θ, the direction of the hydron, is in the direction of $\vec{m}$ with respect to the positive x-axis, and u_m is the component of the current in the direction of $\vec{m}$. The ray curvature of the hydron is defined by

$$\frac{d\theta}{ds_m} = \frac{1}{G'}\left[\sin\theta \frac{\partial}{\partial x}(G' + u_m) - \cos\theta \frac{\partial}{\partial y}(G' + u_m)\right] \tag{49}$$

where ds_m is an arc length along the trajectory of an orthogonal to a hydron crest (interference maximum of wave packet). This result is comparable to Snell's law (23). In the absence of currents (49) reduces to the result presented by Breeding (1981).

5.3 Ray curvature for rays. Equation (14) can be written

$$\vec{G} = G' \frac{\vec{m}}{m} + \vec{u} \tag{50}$$

Consider the derivative $d(m\vec{G})/dt$ where (50) is used.

$$\frac{d}{dt}(m\vec{G}) = G' \frac{d\vec{m}}{dt} + \vec{m}\frac{dG'}{dt} + m\frac{d\vec{u}}{dt} + \vec{u}\frac{dm}{dt} \tag{51}$$

For a wave packet, in place of (39) it follows that

$$\frac{d\vec{m}}{dt} = -m\nabla G' - \vec{m}\cdot\nabla\vec{u} - \vec{m}\times(\nabla\times\vec{u}) \tag{52}$$

For steady state $\partial G'/\partial t = 0$ and $\partial\vec{u}/\partial t = 0$. Therefore, along a ray

$$\frac{dG'}{dt} = \vec{G}\cdot\nabla G' \tag{53}$$

$$\frac{d\vec{u}}{dt} = \vec{G}\cdot\nabla\vec{u} \tag{54}$$

When (50) is substituted into (54), and only the first order term in $\vec{u}$ is retained, it is found that

$$\frac{d\vec{u}}{dt} = \frac{G'}{m}\vec{m}\cdot\nabla\vec{u} \qquad (55)$$

The substitution of (52), (53), and (55) into (51) yields

$$\frac{d}{dt}(m\vec{G}) = -G'm\nabla G' - G'\vec{m}\times(\nabla\times\vec{u}) + \vec{m}(\vec{G}\cdot\nabla G') + \vec{u}\frac{dm}{dt} \qquad (56)$$

Since $u \ll G$, and by neglecting terms higher than first order in u, an approximation to (56) is obtained

$$\frac{d}{dt}(m\vec{G}) = -G\,m\nabla G' - mG\hat{e}_r\times(\nabla\times\vec{u}) + m\hat{e}_r(\vec{G}\cdot\nabla G') + \vec{u}\frac{dm}{dt} \qquad (57)$$

where the unit vector $\hat{e}_r = \vec{G}/G$.

Also

$$\frac{d}{dt}(m\vec{G}) = \hat{e}_r\frac{d}{dt}(mG) + mG\frac{d\hat{e}_r}{dt} \qquad (58)$$

where $\hat{e}_r$ and $d\hat{e}_r/dt$ are perpendicular. A unit vector normal to $\hat{e}_r$ will be denoted by $\hat{n}_r$. Equations (57) and (58) can be combined to obtain

$$\frac{d\hat{e}_r}{dt} = -\nabla G' - \hat{e}_r\times(\nabla\times\vec{u}) + \frac{\vec{u}}{mG}\frac{dm}{dt} + \hat{e}_r\left[\frac{1}{G}(\vec{G}\cdot\nabla G') - \frac{1}{mG}\frac{d}{dt}(mG)\right] \qquad (59)$$

The third term on the right hand side of the equation can be neglected since $u \ll G$ and since the wave properties are assumed to change little over distances of one wavelength.

The direction of the ray, i.e., the direction of $\hat{e}_r$ with respect to the positive x-axis is denoted by ρ. Then

$$\frac{d\rho}{dt} = \hat{n}_r\cdot\frac{d\hat{e}_r}{dt} = -\hat{n}_r\cdot\nabla G' - \hat{n}_r\cdot[\hat{e}_r\times(\nabla\times\vec{u})] \qquad (60)$$

For surface water waves only the vertical component $\vec{\zeta}$ of the vorticity affects the ray trajectory. The z-component of $(\nabla \times \vec{u})$ is given by

$$\vec{\zeta} = \hat{k}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) \qquad (61)$$

Simplification of (60) leads to

$$\frac{d\rho}{dt} = \sin\rho\frac{\partial G'}{\partial x} - \cos\rho\frac{\partial G'}{\partial y} + \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \qquad (62)$$

Further, since $ds_r \approx G'\,dt$, where ds_r is an element of arc length along the ray, it is found that

$$\frac{\partial \rho}{\partial s_n} = \frac{1}{G'}\left(\sin\rho\,\frac{\partial G'}{\partial x} - \cos\rho\,\frac{\partial G'}{\partial y} + \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) \tag{63}$$

This is the ray curvature expression for the rays. If the currents vanish (63) reduces to the result obtained by Breeding (1981).

6. Discussion

Equation (63) defines the trajectories of rays, and yields the paths of constructive interference of waves (wave packets) accounting for the refraction effect of bottom topography and currents. At each point along a ray the packet direction θ is determined by (49) and the wavelet direction γ is determined by (47). The behavior of the waves is more complicated if there is dispersion than if there is not.

The phase speed v and conventional group speed $U = \partial\omega/\partial k$ of a surface gravity water wave for water of arbitrary depth h are given by (Lamb, 1932)

$$v = \left(\frac{g}{k}\tanh kh\right)^{\frac{1}{2}} \tag{64}$$

$$U = \frac{v}{2}\left(1 + \frac{2kh}{\sinh 2kh}\right) \tag{65}$$

If there are no currents the velocities $\vec{U}$ and $\vec{v}$ are in the same direction (Phillips, 1977; Breeding, 1978). If the waves are carried by a current then $\vec{U}'$ and $\vec{v}'$ are in the same direction but $\vec{U}$ and $\vec{v}$ are not.

Under certain conditions (64) and (65) can be simplified. If $\tanh kh = 1$, a condition which defines deep water, then

$$v = \left(\frac{g}{k}\right)^{\frac{1}{2}} \tag{66}$$

$$U = \frac{v}{2} \tag{67}$$

Shallow water can be defined by the condition tanh kh = kh. In this event

$$U = v = (gh)^{\frac{1}{2}} \tag{68}$$

In shallow water the waves are not dispersive. For all other water depths the waves are dispersive.

6.1 No dispersion. Without dispersion (14) becomes

$$\vec{G} = v'\hat{e}_k + \vec{u} \tag{69}$$

The ray curvature expressions (47) and (49) are equivalent, i.e., $\theta = \gamma$. However, the rays are not normal to the wave fronts, i.e., $\rho \neq \gamma$. Arthur (1950) considered the refraction of shallow water waves moving in a current over a topography with variable water depth. In the event there are no currents the ray curvature expressions (47), (49), and (63) are identical. Then $\rho = \theta = \gamma$. If in addition v is constant γ does not change.

6.2 Dispersion. When there is dispersion and refraction occurs due to both bottom topography and currents the directions ρ, θ, and γ are all different. There have been several investigations in which refraction is considered due to only bottom topography or only currents. If there aren't any currents the ray curvature expressions (49) and (63) are equivalent.

Johnson (1947) used (19) to determine the angle of refraction for deep water waves arriving at an angle to a linear current. In deep water there is no refraction due to bottom topography. Only the change in the direction of the wavelets was considered.

Kenyon (1971) considered the refraction of wave packets by currents in deep water. He used Johnson's (1947) refraction result to obtain the direction of the wavelets. Ray trajectories were determined using the ray curvature equation

$$\frac{\partial \rho}{\partial s_n} = \frac{\kappa}{U'} \qquad (70)$$

This result follows from (63) if only the current refraction terms are retained and $G' = U'$. Kenyon's method was also used by Teague (1974).

Independently of Kenyon (1971), Breeding (1972) used what is essentially the same method to determine the refraction of wave packets by bottom topography. The wavelet direction was determined using Snell's law with phase velocity. The ray trajectories were determined using the ray curvature equation

$$\frac{\partial \rho}{\partial s_n} = \frac{1}{U}\left(\sin\rho \frac{\partial U}{\partial x} - \cos\rho \frac{\partial U}{\partial y}\right) \qquad (71)$$

where $\theta = \rho$. As does (70), this equation follows from (63) if $G = U$, except that the refraction terms involving bottom topography are retained instead of the terms in $\vec{u}$.

The refraction method used by Kenyon (1971) leads to a prediction of different directions for $\vec{U}' = \vec{U} - \vec{u}$ and $\vec{v}'$, whereas Breeding (1972) obtains different directions for $\vec{U}$ and $\vec{v}$. These results are inconsistent with the definitions of the conventional group velocity and phase velocity for surface water waves. However, the trajectories obtained in this fashion can be a good approximation to those determined using (63). This is the case for refraction due to bottom topography if $\phi = \theta - \gamma$ has small or moderate values. It is also the case for refraction due to currents if additionally $\rho \approx \theta$ where θ is determined by (49).

When refraction occurs due only to bottom topography (47) reduces to the result obtained by Munk and Arthur (1952) and Arthur <u>et al.</u> (1952) for waves moving with phase velocity. This ray curvature expression has been widely used to determine the trajectories of hydrons on the assumption that the hydrons follow the same paths as monochromatic waves. However, these

trajectories do not coincide with the paths of constructive interference which are obtained by the superposition of individually refracted sine waves to construct wave packets.

Considering refraction due to bottom topography, Breeding (1978) used (63) to determine wave packet trajectories. The wavelet directions were determined using Snell's law with phase velocity.

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Refraction equations are derived for hydrons (wave packets) in which the water depth and current are both variable. A comparison is made of the refraction equations obtained by different investigators.

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